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Generalização da volatilidade implícita de opções aplicada a ações.

Generalization of the Implied Volatility of Options Applied to Shares.

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Abstract

Background: The historical volatility of assets has been widely studied and used in financial markets. In addition, the implied volatilities of options are increasingly being used as possible predictors of future volatility. Although the market works by pricing future expectations, there is no formal definition in the literature of what we call the implied volatility of a stock.
Objective: To generalize the concept of an option's implied volatility and derive a new volatility to be assigned to the underlying stock, now named "asset implied volatility".
Methods: The implied volatilities of options were calculated within the Black-Scholes model using historical stock and option prices, as well as characteristics such as expiration date, strike price, and others. The theoretical option price was calculated using variables extracted from the market, except for implied volatility, which was calculated by numerically inverting the price equation. Option's greeks were also calculated and used to calculate the weighted average of the implied volatilities. Historical volatility was calculated using the Yang-Zhang method and was used for comparison.
Results: The "implied volatility of the asset" showed no correlation with the asset's historical volatility. This new implied volatility, assigned to the asset, suggests that this metric may contain additional information since it does not incorporate information already measured in another way.
Relevance: The results have a potential contribution to economic agents as they can be used as an additional tool in financial resource allocation strategies for investment or asset trading.

Keywords: Implied Volatility; Historical Volatility; Stocks; Options; Derivatives.

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1 Introduction

For many years, researchers have been interested in predicting the returns on financial assets (Migliato, Storani & Masili, 2008). Recently, this interest has turned to forecasting volatility. The changes observed in stock market returns in recent years have highlighted the need for and importance of risk management. Since then, many studies have been published and several risk measurement tools have been developed (Laplante, Desrochers & Préfontaine, 2008).

Volatility forecasting has several applications in the fields of finance and ultimately in macroeconomics. In option pricing models, volatility forecasting can help determine the proportion of investment in a stock portfolio. Given that the volatility of a financial asset's future returns, measured through the variance or standard deviation of returns, is an essential component of portfolio selection, this measure should be as accurate as possible (Laplante, Desrochers & Préfontaine, 2008).

An option pricing model requires several variables as inputs: underlying asset price (spot), strike price, time remaining to exercise (expiration), interest rate, and volatility. Given an option pricing model, the idea of implied volatility (IV) is to input parameters collected from the market such that the option price can be interpreted as a function of volatility alone. By imposing equality between the price provided by the model (called the theoretical price) and the option price in the market (premium), we can compute the value of expected or implied volatility (Hansen, 2001). Generally, the market price of an option differs from its theoretical price, also known as the fair price (Kaeppel, 2002).

Option prices are highly correlated with market expectations of the future value of the underlying asset. Under rational assumptions, markets use all available information to shape their expectations of future volatility. Thus, the options market price reveals the true estimate of volatility. Furthermore, if markets are efficient, the market expectation, i.e., the implied volatility, is the best possible forecast given the current information available (Hansen, 2001). That is, all the information needed to explain future volatility generated by all other market variables must be inferred by implied volatility (Poterba & Summers, 1986; Sheikh, 1989).

Implied volatility in an option price is widely regarded as the options market's prediction of future volatility returns over the remaining time of the option (Christensen & Prabhala, 1998). The hypothesis that implied volatility is a rational forecast, assumed from perceived volatility, has been frequently tested in the literature (Harvey & Whaley, 1992; Canina & Figlewski, 1993).

Therefore, option pricing occurs through the evaluation of financial operators in relation to the future volatility of assets. Thus, in the hypothesis that options markets are efficient and the pricing model is adequate, all relevant information is contained in the option premium. Therefore, implied volatility is a good indicator of future volatility (Mello, 2009). For example, Christensen & Prabhala (1998) used S&P 100 options to analyze implied volatility from 1983 to 1995, which gives good statistical confidence to the work. They concluded that implied volatility was a good predictor of future volatility, especially after the 1987 crash.

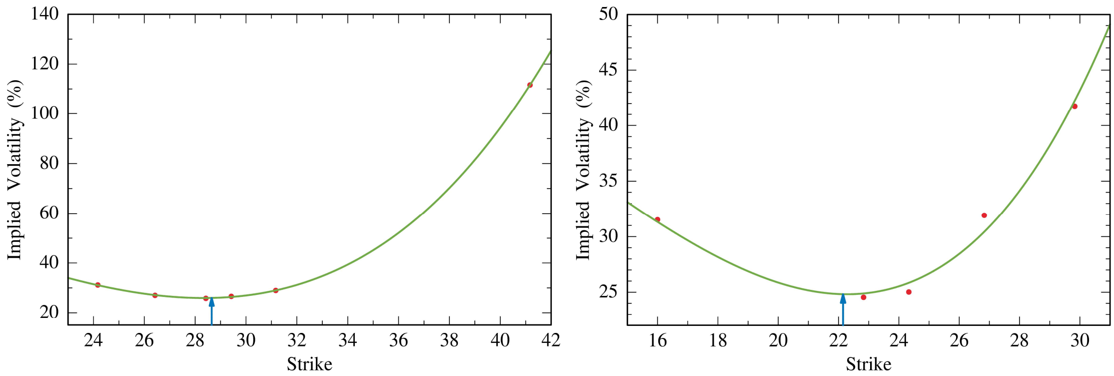
The Brazilian options market has good liquidity only in a few expiration periods and strikes, and they are mostly call options. Put options have lower liquidity. However, the derivative market has been growing annually in terms of the number of offers and liquidity.

One option pricing model that has become well-known is the binomial model, which uses a complex series of iterative calculations to arrive at its version of the fair price of a given option. However, the best-known and most widely used pricing model for derivatives, especially options, was formulated by Fisher Black and Myron Scholes in 1973 and named after the authors.

Within its assumptions, the Black-Scholes (B&S) model predicts that the IV of options on a given asset with the same maturity is constant as a function of strike. However, this is not the case in reality. Markets can present jumps or even extreme events that tend to create inconsistencies in overall trends. For this reason, options with strikes far from the spot price tend to reflect greater market uncertainty, thereby raising their respective premiums. To accommodate these higher premiums in the B&S model, IV grows as the strike moves away from the spot price. This generates a picture of an IV × strike that typically resembles a smile, and for this reason, this behavior is called a volatility smile.

Figure 1 shows a typical example of the actual values for Vale and Petrobras, where the smallest values for IV are close to the spot price, indicated by the blue arrow.

Figure 1 – Typical examples of implied volatilities of calls for VALE5 (left) and PETR4 (right) showing the so-called volatility smile for options of the same maturity



Legend: The blue arrow indicates the spot price.

Source: Prepared by the authors (2024).

There are several option pricing models, but the fair prices found by most are very close. This is probably due to the fact that many of these models use the same variables to calculate the fair price, also called the theoretical option price. These variables are:

1. Current price of the underlying asset, called the spot price;
2. Strike price of the option under analysis.
3. Current basic interest rates;
4. Number of days remaining until options expire
5. Dividends paid;
6. Historical volatility. However, due to distortions between the fair price and market price (premium), this volatility becomes implied volatility.

Thus, the option premium, as it depends on each of these variables, has a certain sensitivity to the variation in the respective parameters. The sensitivity of the B&S formula to each variable is expressed in terms of the partial derivatives of the premium with respect to each variable in the model, and has important consequences for investments. These respective partial derivatives (first-order and higher-order) are called the "greeks" of the options. The main and most commonly used in the options market are delta, gamma, theta, and vega. Delta shows the sensitivity of the premium with respect to spot price variations, i.e., it is the partial derivative of the premium with respect to the underlying stock price. Gamma is the variation of delta with respect to spot, i.e., it is the second-order derivative of the premium. Theta is the sensitivity of the premium with the passage of time, and vega (which is not a Greek letter) is the rate of change in the premium with respect to the volatility of the underlying asset.

Historical volatility (HV), also known as realized or past volatility, is often used in asset allocation strategies in the financial market and has a variety of calculation methods that exhibit multiple approaches. Just to name a few examples, one can compute historical volatility by the standard deviation of logarithmic returns using a variety of GARCH family methods, stochastic models, and Yang and Zhang's method (2000), among others. So-called volatility traders, i.e., financial market agents who trade the volatility of an asset rather than its price (Sinclair, 2008; Bennett, 2014), use HV when dealing with stocks. When traded assets are derivatives (calls or puts), IV is traditionally used. The concept of IV for stocks does not appear in this scenario, or at least, there are no known published strategies in this regard. It is worth noting that many strategies are closed and proprietary, as there is great economic interest in this area. Thus, this study aims to generalize the concept of IV beyond the derivative market. Although conceptually general, this study uses as application examples the common shares (PN) of Petrobras (PETR4) and Vale (VALE5), two large companies traded on the São Paulo Stock Exchange (B3). In addition to generalizing the concept of IV for stocks, this study proposes a method for calculating the IV for stocks, connecting it to the IV of the respective options. This generalization is useful in adding new information to asset volatility analyses, offering an additional analysis tool for financial agents.

The main advancement is the calculation of a new metric (or indicator) that aims to add information regarding future volatility assessed by the market for a given stock. This volatility estimator is common for derivatives, but this concept is not applied to stocks. Options are dependent on the asset price in the spot market and other market variables, and have a much more complex behavior. With this proposal, we intend to transfer this complexity to the stock market.

2 Method

2.1 Database and computer codes

The database was purchased (with own budget) from the quote provider QuotesBr (http://www.quotebr.com/) containing the historical series of VALE5 and PETR4 shares, in addition to the series of all respective options, traded in the period from 09/12/2011 to 01/04/2016. The timeframe of the database was daily and contained open, low, high, and close prices (end-of-day), number of trades, number of shares traded, and financial volume for the two chosen assets. For the respective options, in addition to the aforementioned data, the database contains strike price and expiration date. Intraday data were not considered because they were not relevant for this research. The historical series of the risk-free interest rates (SELIC), necessary for the calculation of the theoretical price of the options and the implied volatility, were collected from the website of the Brazilian Central Bank (Bacen) at https://www.bcb.gov.br/controleinflacao/historicotaxasjuros.

For all calculations, which will be described in the next section, we developed our own computer code in FORTRAN language and used the Intel Parallel Fortran XE 2013 SP1 compiler. This choice allows for the generation of an executable program (.exe) with a substantial increase in processing speed, which can be easily ported to other computer systems.

2.2 Mathematical development

As outlined in the previous section, each option on a given asset, whether a call or put, has its own implied volatility. The premium (market price) of the option under the Black-Scholes model depends on the six factors already listed, of which the first five are known. Thus, a certain volatility must be given, so that the theoretical option price equals the market price. This procedure does not exist for stocks because, unlike derivatives, which are put and call contracts, stocks have no expiration date, and it is the price variation that determines their volatility, so there is no future (implied) volatility. Thus, the generalization proposed in this paper associates the implied volatility of the options of an asset traded on a given market day with the respective underlying asset. In general, for a given asset, the implied volatilities of the options are calculated, and their arithmetic mean is weighted by a function given by the modulus of the product of the greeks by the financial volume, number of trades, and number of days to expiration of each option. This weighting aims to give more weight to at-the-money (ATM) options while also considering options with longer maturities and decreases the weight of those that are very close to expiration and that, for this reason, may present price distortions and, consequently, volatility. In addition, it gives more weight to options with higher trading volumes, which may occur for non-ATM options according to the expectations of market agents. This weighted average of the IV's is assigned to the asset as if it were its future volatility. The calculation procedure is outlined in detail below.

Initially, the historical volatility of assets was calculated using the method of Yang & Zhang (2000), as it is the most efficient among the six most widely used methods (Bennett, 2014, p. 232). In this way, one can compare the HV with the IV of the asset in question. The Yang-Zhang historical volatility estimator is given below:

$$\sigma_{YZ}^2 = \sigma_{CO}^2 + k\sigma_{OC}^2 + (1-k)\sigma_{RS}^2, \tag{1}$$

in which

$$\sigma_{CO}^2 = \frac{F}{N-1}\sum_{i=1}^{N}\left[\ln\left(\frac{o_i}{c_{i-1}}\right) - \overline{\ln\left(\frac{o_i}{c_{i-1}}\right)}\right]^2, \tag{2}$$

$$\sigma_{OC}^2 = \frac{F}{N-1}\sum_{i=1}^{N}\left[\ln\left(\frac{c_i}{o_i}\right) - \overline{\ln\left(\frac{c_i}{o_i}\right)}\right]^2, \tag{3}$$

$$\sigma_{RS}^2 = \frac{F}{N}\sum_{i=1}^{N}\left[\ln\left(\frac{h_i}{c_i}\right)\ln\left(\frac{h_i}{o_i}\right) + \ln\left(\frac{l_i}{c_i}\right)\ln\left(\frac{l_i}{o_i}\right)\right] \tag{4}$$

and

$$k = \frac{\alpha - 1}{\alpha + \frac{N+1}{N-1}}. \tag{5}$$

In the above equations, o_i, l_i, h_i, and c_i are the open, low, high, and close prices of the asset for the i-th day, respectively. N is the sample size and $F = 252$ is the constant used to annualize the result, which is the average number of business days in the year. The constant k [equation (5)] used in equation (1) is obtained to minimize the variance given by equation (1) and depends on the constant α, which originates from Brownian motion with drift. Yang & Zhang (2000) estimate the value of $\alpha = 1.34$.

The Black-Scholes option pricing model determines the fair price of options using the following equations:

$$C = S\Phi(d_1) - \Phi(d_2)Ke^{-r\Delta T} \tag{6}$$

and

$$P = -S\Phi(-d_1) + \Phi(-d_2)Ke^{-r\Delta T}, \tag{7}$$

in which

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\Delta T}{\sigma\sqrt{\Delta T}}, \tag{8}$$

$$d_2 = d_1 - \sigma\sqrt{\Delta T}, \tag{9}$$

and

$$\Phi(z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{-x^2/2}dx. \tag{10}$$

In these equations, we have the following:

- C: call option premium
- P: put option premium
- S: spot price of the underlying asset
- K: option strike
- ΔT: time remaining until expiration of the option (in years)
- r: risk-free interest rate (SELIC)

- Φ : standard normal cumulative distribution function
- σ: annualized volatility of the underlying asset

Using these equations, we can calculate the value of the volatility σ. However, it is not possible to obtain the value of σ analytically, i.e., by isolating it from the equations of the model, because these equations are not invertible. As there is no closed-form solution for the inverse problem involved, one must resort to numerical root-finding methods for the solution. Any minimization method can be used, such as Newton-Raphson, Brent, Simplex, Regula-Falsi, or other known methods. In this study, a fast-converging iterative method (Płuciennik, 2007) was used, as explained below. First, we compute a zero-order estimate of volatility σ using the following equation:

$$\sigma_0 = \sqrt{2\frac{\left|\ln\left(\frac{S}{K}\right) + r\Delta T\right|}{\Delta T}}. \tag{11}$$

Then, equation (12) is iterated until the desired convergence is achieved, i.e.,

$$\sigma_{n+1} = \sigma_n - |X - X_n|\frac{e^{d_1^2/2}}{S}\sqrt{\frac{2\pi}{\Delta T}}, \tag{12}$$

where $n = 0, 1, \cdots, n_{\max}$. X is the option premium that is being traded in the market, X_n is the fair price calculated from equations (6) or (7) respectively, and d_1 is given in equation (8). Equation (12) is iterated until $|X - X_n| < \epsilon$, where ϵ is the desired precision. For our purposes, $\epsilon <$ R\$ 0.002 was considered. Under these circumstances, σ calculations converge quickly, usually with only two or three iterations.

After calculating the implied volatilities of all options traded on a given day, we obtain their weighted average, and thus conceptualize this average as the implied volatility of the underlying asset. Thus,

$$\sigma_{\text{Stock},k} = \frac{\sum_{i=1}^{N}\sigma_{i,k}W_{i,k}}{\sum_{i=1}^{N}W_{i,k}}. \tag{13}$$

In this equation, index k indicates the k-th trading day and index i runs through all N options traded on day k for the stock. The IV's $\sigma_{i,k}$ are calculated by equation (12). The weighting function $W_{i,k}$ is given by the absolute value of the product of the greeks by the financial volume, the number of trades and the expiration time of each option, i.e.,

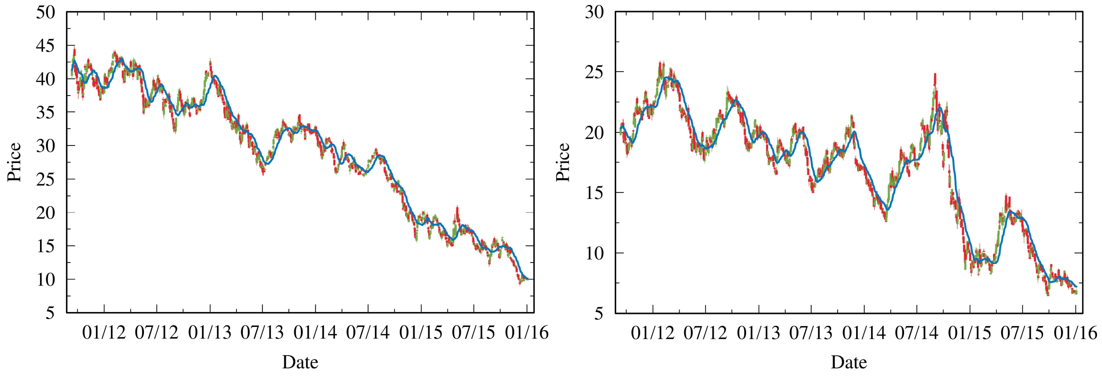
$$W_{i,k} = \left|\delta_{i,k}\Gamma_{i,k}\theta_{i,k}\mathrm{Vol}_{i,k}\mathrm{Neg}_{i,k}T_{i,k}\right|. \tag{14}$$

Because of typical market distortions, some constraints must be applied to the use of options traded on the day. For example, even if there are trades, options that are deep in the money (ITM) or too out of the money (OTM), that is, with strikes that are too far away from the spot price, usually have distortions in the value of the premium traded, even when considering the smile effect (see Figure 1), producing a result that is also distorted for the respective implied volatility. This is due to the low liquidity of these options, as there are not enough players to equalize the market. Therefore, options with these characteristics are disregarded when calculating the average. Two parameters were used as exclusion criteria for each option: a) minimum number of trades equal to 10, and b) strike value within a range of $\pm 15\%$ of the underlying asset price on the respective day.

3 Results and Discussion

This section presents the results for Petrobras (PETR4) and Vale (VALE5) preferred shares for the period from 09/12/2011 to 01/04/2016, summing 1065 trading days. Given the restrictions outlined in the previous section, data from 9466 VALE5 options traded in the period were used. For PETR4, 7182 options were used for the same period. As an example, Figure 2 shows the time series of the prices charged for the Vale and Petrobras shares in this period, showing a loss of approximately 77% of their market value. Also shown is the curve of the simple moving average (SMA) of 22 periods, which is the number of business days in a month. This average shows smoothing of the time series and allows easier identification of possible trends. However, it should be noted that SMA is a lagging indicator that shows long-term trends and is therefore not always useful for indicating moments of market entry and exit. Typically, periods of large declines exhibit high volatility due to risk aversion. On the other hand, periods of increasing volatility present good opportunities for volatility trades, regardless of the price of the asset in question. During the same period, Petrobras exhibited a similar behavior.

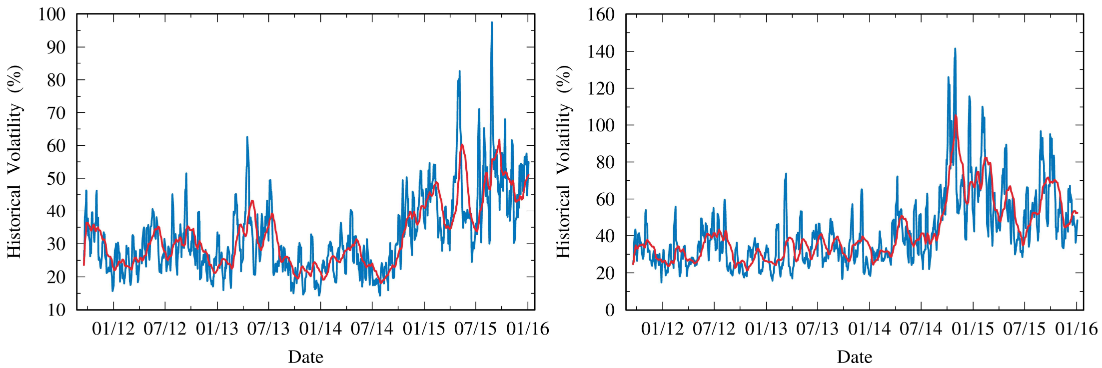
Figure 2 – Historical series of the market prices of VALE5 (left) and PETR4 (right), and the simple moving average of 22 periods



Source: Prepared by the authors (2024).

Figure 3 shows the results for the historical, or realized, volatility of VALE5 and PETR4, using the Yang-Zhang method, according to equations (1) – (5).

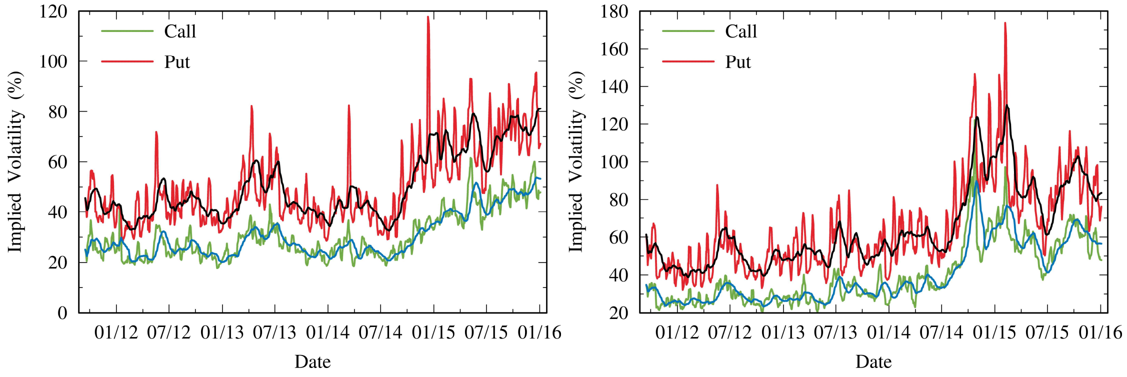
Figure 3 – Historical volatility of VALE5 (left) and PETR4 (right), and the 22-period simple moving average



Source: Prepared by the authors (2024).

Figure 4 shows the weighted average implied volatilities according to the proposal of this study. As an example, the average IV's of calls and puts are shown separately. However, for the purposes of this paper, the average of both will be used from this point on.

Figure 4 – Comparison of the weighted average implied volatilities of calls and puts for VALE5 (left) and PETR4 (right)

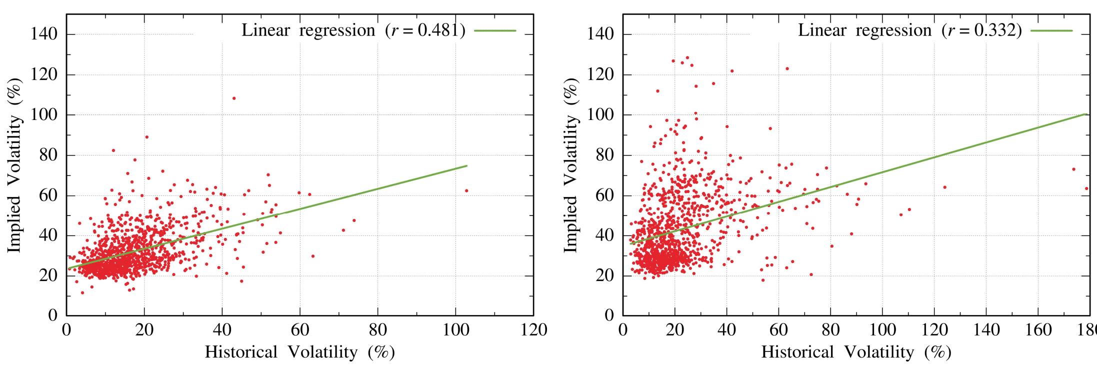


Source: Prepared by the authors (2024).

In Figure 4, it can be seen that over the entire period, the average IV of the puts is higher than that of calls. This is compatible with the great period of fall in asset value, because the puts, which are put options, have increased liquidity and gain greater market interest, especially from hedge funds and portfolio protection.

To show that the IV of the stock, as proposed in this study, can be an additional tool in volatility buying and selling trades, Figure 5 shows a correlation analysis between the HV and the IV of the stock.

Figure 5 – Correlation between historical and implied volatilities of VALE5 (left) and PETR4 (right)



Source: Prepared by the authors (2024).

One can see that the least-squares regression analysis provides correlation coefficient values of $r = 0.481$ for VALE5 and $r = 0.332$ for PETR4. This shows that these two volatilities can deliver different information because they do not depend on each other. In fact, at least conceptually, HV shows the past, while IV shows an expectation of future volatility, according to financial market agents.

4 Conclusions

This study proposes a generalization of the concept of implied volatility, widely used in derivative transactions, to the respective underlying assets, for which the analysis of realized or historical volatility is exclusively used. This proposal of implied volatility for stocks can be an additional indicator to historical volatility because, in principle, it adds the expectation of future volatility to the asset in question, allowing the formation of a more complete picture of the analysis. This new implied volatility assigned to the underlying asset can even be a substitute for historical volatility in financial traders' strategies.

The "IV of the equity" can be used, for example, by evaluating its difference from the HV. In this case, capital market operators guided by volatility can substitute HV for the aforementioned difference. However, a more detailed analysis requires considering not only the difference between IV and HV, but also the history of both, that is, how close the current IV and HV are to their respective historical minima and maxima (e.g., in the previous year).

While the database used in this investigation may appear outdated, its current state does not substantially impact the present study, given that it introduces a pioneering method for evaluating implicit volatility in financial markets, thereby presenting a wide range of opportunities for analyzing the combination of these volatilities. This, in turn, lays the groundwork for future investigations.

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